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Generalized Laplace - Mellin Integral Transformation

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ABSTRACT: The main propose of this paper is to generalized Laplace-Mellin Integral Transformation in between the positive regions of real axis. We have derived some new properties and theorems .And give selected tables for Laplace-Mellin Integral Transformation.

I. INTRODUCTION

Integral Transformation recognition is one of the most powerful tool in applied Mathematics for solving initial and boundary value problem . It cover wide range of application in various area of Physics, Electrical engineering , Control engineering ,Optics and Signal processing.

There are already existing several work done on the theory and application of Integral Transformation like Fourier , Laplace , Mellin , Hankel, Hilbert , Stieltjes and many other which are mansion in ‘ Generalized Integral Transformation’ . This paper are re-introduced the two type of integration those are Laplace Transformation and Mellin Transformation. Both have there own importance, but in this paper we deduced the combined relation between Laplace Transformation and Mellin Transformation and explaining the extension of generalized function of Laplace- Mellin Integral Transformation (LMIT).

1.1 FORMAL DEFINATION:

The conventional Laplace - Mellin Integral Transformation (LMIT) is defined as

$$\mathfrak{LM}\,[h(l,m)] = H\,[(l,m)] = \int_0^\infty \int_0^\infty h(l,m)\, e^{-sl}\, m^{p-1}\, dl\, dm$$

where the $f(l,m)$ be the function of suitably restricted conventional function on the real line $0 < l < \infty$ and $0 < m < \infty$. Where $s\, and\, p$ be both are the complex variables and $f(l,m)$ is called the time domain representation of the signal processing.

The function is Mellin transformation is denoted by $\mathfrak{M}$ and Laplace transformation is denoted by $\mathfrak{L}$ .Thus the function is combined denoted operator as $\mathfrak{LM}$ for Laplace Mellin Integral Transformation.

The Double Laplace Transformation (two dimension Laplace Transformation) be denoted as

$$\mathfrak{L}_2\,[f(l,t)] = F\,(s,p) = \int_0^\infty \int_0^\infty f(l,t)\, e^{-sl}\, e^{-pt}\, dl\ \, dt$$

The LMIT can be obtained by applying the change of one variable in Double Laplace Transformation. It generated when we take same time function, let the function $e^{-t} = m$, then $t = -logm$ and $f(-logm) = f(m)$. We also arise this generalized condition from two dimension Fourier Transformation. As the result, the Laplace - Mellin Integral Transformation (LMIT) is of a certain type of generalized function $f(l,m)$ can be defined as the application of $f(l,m)$ to the kernel $e^{-sl}\, m^{p-1}$

$$H\,(l,m) = < h(l,m)\ , e^{-sl}\, m^{p-1} >$$

1.2 TABLES FOR SELECTED LAPLACE ELLIN INTEGRAL TRANSFORMATION

NO	$f(l,m)$	$\mathfrak{LM}\,[f(l,m)] = \int_0^\infty \int_0^\infty f(l,m) e^{-sl}\, m^{p-1} dldm$
1	$\frac{1+l}{1+m}$	$\frac{s+1}{s^2}\ \mathcal{T}(p)\ \mathcal{T}(1-p)$
2	$\left(\frac{1+l}{1+m}\right)^n$	$\left(\frac{1}{s}+\frac{n}{s^2}+\frac{n(n-1)}{s^3}+\cdots+\frac{\mathcal{T}(n+1)}{s^{n+1}}\right)\frac{\mathcal{T}(p)\mathcal{T}(n-p)}{\mathcal{T}(n)}$
3	e^{lm}	$\pi\, s^{p-1}\cot(\pi p) \qquad 0 < Re\ p < 1$
4	e^{-lm}	$\pi\, s^{p-1}\operatorname{cosec}(\pi p) \qquad 0 < Re\ p < 1$
5 6	$\frac{e^l-1}{e^m-1}$ $\frac{e^l+1}{e^m+1}$	$\frac{1}{s(s-1)}\ \mathcal{T}(p)\ \zeta(p)$, $\zeta(p)$ is Riemann Zeta function $\frac{2s-1}{s(s-1)}\ (1-2^{1-p})\,\mathcal{T}(p)\ \zeta(p)$
7	$\sin lm$	$\frac{\pi}{2}\ s^{p-1}\ cosec\ \frac{\pi(p+1)}{2}$
8	$\cos\ lm$	$\frac{\pi}{2}\ s^{p-1}\ cosec\ \frac{\pi p}{2}$
9	$\sin(al \pm bm)$	$\frac{b^{-p}\,\mathcal{T}(p)}{s^2+a^2}\ \left(a\, cos\frac{\pi p}{2} \pm s\, sin\frac{\pi p}{2}\right)$
10	$\cos(al \mp bm)$	$\frac{b^{-p}\,\mathcal{T}(p)}{s^2+a^2}\ \left(s\, cos\frac{\pi p}{2} \pm a\, sin\frac{\pi p}{2}\right)$

II. GENERALIZED LAPLACE - MELLIN INTEGRAL TRANSFORMATION

2.1 **DEFINATION :** The Generalized Laplace - Mellin integral transformation (GLMIT) with parameter $\boldsymbol{\varpi}$ of $h(l,m)$ denote by $\mathfrak{LM}\,[h(l,m)]$ performs a liner operation , given by the Integral Transformation .

$$\mathfrak{LM}\,[h(l,m)] = \int_0^\infty \int_0^\infty h(l,m)\ K_\varpi(l,s,m,p)\, dl\, dm = H_\varpi(s,p)$$

The function $K_\varpi(l,s,m,p)$ is called the *Kernel of the Transformation* .Where 's' and 'p' are parameters (may be real or complex) which are independent of 'l' and 'm' .

$$K_\varpi(l,s,m,p) = e^{-sl}\ e^{(\pi p + \varpi^2 \log m - \varpi\ \log l)}\ m^{P-1}$$
$$= e^{(\pi p + \varpi^2 \log m - \varpi\ \log l) - sl}\ m^{P-1}\ ,$$

where $\varpi = \left(1/_2, 0\ -1, -2\ , \dots, -n\ \right) P + \varpi^2 = p$

2.2 TESTING FUNCTION SPACE $\mathfrak{LM}_{a,b,c,d}^{w,x,y,z}$:

Let we define be an open set $\mathfrak{I}$ which be as $(0,\infty)$. In this section we examining the testing function space of $\mathfrak{LM}_{a,b,c,d}$, where $a,b,c,d,l,m \in R^1$ and $s,p \in C^n$. As we kwon that any space is say to be a testing function space on $\mathfrak{I}$ if it satisfied the following condition :

1. The space should be consists entirely of smooth complex value function on $\mathfrak{I}$.
2. The space is must be either a complete countably normed or a complete countably union space.
3. If possible to be define any sequence as $\{\ \phi_t\ \}_{t=1}^\infty$, which is converges in zero element of that space and also their exist for all non negative values of $k \in R^n$, $\{\ \mathfrak{D}^k\ \phi_t\ \}_{t=1}^\infty$ converges to the zero function uniformly on every compact subset of $\mathfrak{I}$.

Let us we define a function $\phi(l,m)$ on the linear space $\mathfrak{LM}_{a,b,c,d}$, where $-\infty < l < \infty$ and $0 < m < \infty$. In this integral we define two function $\mathcal{J}_{a,b}(l)$ and $\lambda_{c,d}(m)$

$$\mathcal{J}_{a,b}(l) \triangleq \begin{cases} e^{al} & 0 \le l < \infty \\ e^{bl} & -\infty < l < 0 \end{cases}$$

and
$$\lambda_{c,d}(m) \triangleq \begin{cases} m^{-c} & 0 < m \le 1 \\ m^{-d} & 1 < m < \infty \end{cases}$$

In their both the function $\mathcal{J}_{a,b}(l)$ and $\lambda_{c,d}(m)$ satisfied the first condition of smooth function because both function has R^n or C^n domain and range will be defined in one dimension real or complex number . Also both function are individual infinitely differentiable function in infinitely real or complex number (R^∞ or C^∞) .

So we can define $\mathfrak{LM}_{a,b,c,d}$ is a linear space with holding properties of addition and multiplication by complex numbers . And also $\mathfrak{LM}_{a,b,c,d}$ define the space of all complex valued smooth functions .

$$\gamma_{j,k}\,\phi(l,m) \triangleq \gamma_{a,b,c,d,j,k}\phi(l,m) \quad \triangleq \sup_{\substack{-\infty<l<\infty \\ 0<m<\infty}} \left|\mathcal{J}_{a,b}(l)\,\lambda_{c,d}(m)\,\mathfrak{D}_l^j\ \mathfrak{D}_m^k\,\phi(l,m)\right| < \infty$$

for each $j\,,k = 0\,,1\,,2\,,\dots\dots\dots\dots\dots$

Where we define a collection of countable seminorm $\{\gamma_{j,k}\}_{\substack{j=0\\k=0}}^{\infty}$ on the linear space $\mathfrak{LM}_{a,b,c,d}$. We used that notation $\gamma_{j,k}$ in place of $\gamma_{a,b,c,d,j,k}$. Again $\gamma_{0,0}$ be the norm on $\mathfrak{LM}_{a,b,c,d}$ which is the zero element of $\mathfrak{LM}_{a,b,c,d}$. From those above definition we can called $\{\gamma_{j,k}\}_{\substack{j=0\\k=0}}^{\infty}$ the countable multinorm and the linear space $\mathfrak{LM}_{a,b,c,d}$ is called countably multinormed space on $\mathfrak{I}$.

For each fixed nonnegative numbers j and k , we define a Cauchy sequence $\{\phi_t\}_{t=1}^{\infty}$ in $\mathfrak{LM}_{a,b,c,d}$ which is converges ϕ_t in $\mathfrak{LM}_{a,b,c,d}$ which is defined in the equation . By the definition we can say that $\mathcal{J}_{a,b}(l)\,\lambda_{c,d}(m)\,\mathfrak{D}_l^j\ \mathfrak{D}_m^k\,\phi(l,m)$, this function be uniformly convergent on $-\infty < l < \infty$ and $0 < m < \infty$ as $t \to \infty$. So by theorems we can say the function $\phi_t(l,m)$ is also convergent $\phi(l,m)$ as $t \to \infty$. Again for any $\epsilon > 0$ there exist an $H_{j,k}$ such that , for every $t, \mu > H_{j,k}$

$$\left|\mathcal{J}_{a,b}(l)\,\lambda_{c,d}(m)\,\mathfrak{D}_l^j\ \mathfrak{D}_m^k\left[\phi_t(l,m) - \phi_\mu(l,m)\right]\right| < \epsilon$$

again for all l and m ,we taking the limit as $t \to \infty$. So the function reduced its form and we get $\left|\mathcal{J}_{a,b}(l)\,\lambda_{c,d}(m)\,\mathfrak{D}_l^j\ \mathfrak{D}_m^k\left[\phi_t(l,m) - \phi(l,m)\right]\right| \le \epsilon \quad -\infty < l\,, m < \infty\ ,\ t > H_{j,k}$

Due to as we know for each j, k .The seminorm $\gamma_{j,k}(\phi_t - \phi) \to 0$, as $t \to \infty$. So , finally we can say the paritial function $\left\{\mathfrak{D}_l^j\ \mathfrak{D}_m^k\,\phi_t(l,m)\right\}_{t=1}^{\infty}$ is converges to the zero function uniformly on every compact subset of $\mathfrak{I}$. At the last we can say the $\mathfrak{LM}_{a,b,c,d}$ is the testing function space on the open set $\mathfrak{I}$.

2.3 DISTRIBUTIONAL GENERALISED $\mathfrak{LM}_{a,b,c,d}^{w,x,y,z}$: Let $\mathfrak{LM}_{a,b,c,d}^{w,x,y,z\,\ddagger}$ is the dual space of $\mathfrak{LM}_{a,b,c,d}^{w,x,y,z}$ .This space $\mathfrak{LM}_{a,b,c,d}^{w,x,y,z\,\ddagger}$ consist of continuous liner function on $\mathfrak{LM}_{a,b,c,d}^{w,x,y,z}$. The distributional Generalized Laplace - Mellin integral transformation of $h(l,m) \in \mathfrak{LM}_{a,b,c,d}^{w,x,y,z\,\ddagger}$ is defined as

$$\mathfrak{LM}\,[h(l,m)] = H_\varpi(s,p) = \langle h(l,m), K_\varpi(l,s,m,p)\,\rangle$$
$$= \langle h(l,m), e^{(\pi p + \varpi^2 \log m - \varpi\ \log l) - sl}\ m^{P-1}\,\rangle$$

where for each fixed $l, m\ (0 < l < \infty\ and\ 0 < m < \infty)$, $p = P + \varpi^2\ and\ \varpi = \left({}^{1}\!/_{2}\,, 0\ -1, -2\,, \dots, -n\ \right)$, the right hand side of this equation has a sense as an application of $e^{(\pi p + \varpi^2 \log m - \varpi\ \log l) - sl}\ m^{P-1} \in \mathfrak{LM}_{a,b,c,d}^{w,x,y,z}$

III. BACIS PROPERTIES

3.1 LINEAR PROPERTY: When GLMIT is

$$\mathfrak{LM}\,[h(l,m)] = \int_0^\infty\int_0^\infty h(l,m)\ e^{(\pi p + \varpi^2 \log m - \varpi\ \log l) - sl}\ m^{P-1}\,dl\,dm = H_\varpi(s,p)$$

then,

$$\mathfrak{LM}\,[ah(l,m) + bj(l,m)] = \int_0^\infty\int_0^\infty [ah(l,m) + bj(l,m)]\ e^{(\pi p + \varpi^2 \log m - \varpi\ \log l) - sl}\ m^{P-1}\,dl\,dm$$
$$= aH_\varpi(s,p) + bJ_\varpi(s,p)$$

3.2 SCALING PROPERTY: When GLMIT is

$$\mathfrak{LM}\,[h(l,m)] = \int_0^\infty\int_0^\infty h(l,m)\ e^{(\pi p + \varpi^2 \log m - \varpi\ \log l) - sl}\ m^{P-1}\,dl\,dm = H_\varpi(s,p)$$

then,

$$\mathfrak{LM}\left[h\left(\frac{l}{a},\frac{m}{b}\right)\right] = \int_0^\infty\int_0^\infty h\left(\frac{l}{a},\frac{m}{b}\right)\ e^{(\pi p + \varpi^2 \log m - \varpi\ \log l) - sl}\ m^{P-1}\,dl\,dm = H_\varpi(s,p)$$

put $\frac{l}{a} = c \Rightarrow l = ca$ and $\frac{m}{b} = f \Rightarrow m = fb$

differentiating, $dl = a\,dc$ and $dm = b\,df$

$$\mathfrak{LM}\,[h(c,f)] = a^w\ b^p \int_0^\infty\int_0^\infty h(c,f)\ e^{(\pi p + \varpi^2 \log m - \varpi\ \log l) - sl}\ m^{P-1}\,dl\,dm = H_\varpi(s,p)$$

3.3 POWER PROPERTY: When GLMIT is

$$\mathfrak{LM}\,[h(l,m)] = \int_0^\infty \int_0^\infty h(l,m)\ e^{(\pi p + \varpi^2 \log m - \varpi\ \log l) - sl}\ m^{P-1}\, dl\, dm = H_\varpi(s,p)$$

then,

$$\mathfrak{LM}\,[h(l,m^r)] = \int_0^\infty \int_0^\infty h(l,m^r)\ e^{(\pi p + \varpi^2 \log m - \varpi\ \log l) - sl}\ m^{P-1}\, dl\, dm = H_\varpi(s,p)$$

when put $m^r = t \Rightarrow m = t^{\frac{1}{r}}$, then

$$\mathfrak{LM}\,[h(l,t)] = \frac{1}{r}\int_0^\infty \int_0^\infty h(l,t)\ e^{\left(\pi p + \frac{\varpi^2}{r} \log t - \varpi\ \log l\right) - sl}\ t^{\frac{P}{r}-1}\, dl\, dm = H_\varpi\left(s, \frac{p}{r}\right)$$

IV. SOME RESULT

4.1 INVERSION THEOREM: We assume that $\mathfrak{LM}\,[h(l,t)]$ is regular function in the strips $|Re\ s| < c$ and $|Re\ p| < f$ (c and f are real number) of consequently the 's' and 'p' planes with constants 'a' and 'b' as $a - i\infty < s < a + i\infty$ and $b - i\infty < p < b + i\infty$

.

$$\mathfrak{LM}\,[h(l,m)] = \int_0^\infty \int_0^\infty h(l,m)\ e^{(\pi p + \varpi^2 \log m - \varpi\ \log l) - sl}\ m^{P-1}\, dl\, dm = H_\varpi(s,p)$$

Form Laplace and Mellin Transformation, we know the inversion formula

$$\mathfrak{L}^{-1}\{F(s)\} = f(l) = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} e^{sl}\ F(s)\, ds$$

and

$$\mathfrak{M}^{-1}\{F(p)\} = f(m) = \frac{1}{2\pi i}\int_{b-i\infty}^{b+i\infty} m^{-p}\ F(p)\, dp$$

then,

$$h(l,m) = \mathfrak{LM}^{-1}\{H_\varpi(s,p)\} = \frac{1}{(2\pi i)^2}\int_{a-i\infty}^{a+i\infty}\int_{b-i\infty}^{b+i\infty} \left(\frac{\pi}{s}\right)^\varpi \frac{1}{p} e^{(sl+\pi p)}\, m^{-p}\ \ \mathfrak{LM}\,[h(l,m)]\, ds\, dp$$

PROOF :

$$\mathfrak{LM}\,[h(l,m)] = \int_0^\infty \int_0^\infty h(l,m)\ e^{(\pi p + \varpi^2 \log m - \varpi\ \log l) - sl}\ m^{P-1}\, dl\, dm = H_\varpi(s,p)$$

then

$$h(l,m) = \int_0^\infty \int_0^\infty e^{(\pi p + \varpi^2 \log m - \varpi\ \log l) - sl}\, m^{P-1}\left[\frac{1}{(2\pi i)^2}\int_{a-i\infty}^{a+i\infty}\int_{b-i\infty}^{b+i\infty} e^{sl}\ m^{-p}\ \mathfrak{LM}\,[h(l,m)]\, ds\, dp\right] dl\, dm$$

$$= \frac{1}{(2\pi i)^2}\int_{a-i\infty}^{a+i\infty}\int_{b-i\infty}^{b+i\infty} e^{sl}\ m^{-p}\ \mathfrak{LM}\,[h(l,m)]\, ds\, dp\left[\int_0^\infty \int_0^\infty e^{(\pi p + \varpi^2 \log m - \varpi\ \log l) - sl}\ \ m^{P-1}\, dl\, dm\right]$$

$$= \frac{1}{(2\pi i)^2}\int_{a-i\infty}^{a+i\infty}\int_{b-i\infty}^{b+i\infty} e^{sl+\pi p}\ m^{-p}\ \ \mathfrak{LM}\,[h(l,m)]\, ds\, dp\ \ \left(\frac{\pi}{s}\right)^\varpi\ \frac{1}{p}$$

$$= \frac{1}{(2\pi i)^2}\int_{a-i\infty}^{a+i\infty}\int_{b-i\infty}^{b+i\infty} \left(\frac{\pi}{s}\right)^\varpi\ \frac{1}{p}\ \ e^{sl+\pi p}\ m^{-p}\ \ \mathfrak{LM}\,[h(l,m)]\, ds\, dp \qquad \varpi = \frac{1}{2}$$

Where $\lim_{m\to\infty} m^P = 1$

$$= \frac{1}{(2\pi i)^2}\int_{a-i\infty}^{a+i\infty}\int_{b-i\infty}^{b+i\infty} \frac{\Gamma(\varpi+1)}{s^{\varpi+1}}\ \frac{1}{p}\ \ e^{sl+\pi p}\ m^{-p}\ \ \mathfrak{LM}\,[h(l,m)]\, ds\, dp \qquad \varpi = (0,-1,-2,\dots,-n)$$

4.2 CONVOLUTION THEOREM : When GLMIT is

$$\mathfrak{LM}\,[h(l,m)] = \int_0^\infty \int_0^\infty h(l,m)\ e^{(\pi p + \varpi^2 \log m - \varpi\ \log l) - sl}\ m^{P-1}\, dl\, dm = H_\varpi(s,p)$$

then,

$$\mathfrak{LM}\,[h(l,m)\, f(t-l,m)] = \frac{1}{(2\pi i)^2}\int_{a-i\infty}^{a+i\infty}\int_{b-i\infty}^{b+i\infty} \frac{\pi^\varpi}{s^\varpi p}\ e^{sl+\pi p}\, m^{-p}\ \mathfrak{LM}\,[h(l,m)]\ \mathfrak{LM}\,[f(t-l,m)]\, dsdp$$

PROOF :

$$\mathfrak{LM}\,[h(l,m)] = \int_0^\infty \int_0^\infty h(l,m)\ e^{(\pi p + \varpi^2 \log m - \varpi\ \log l) - sl}\ m^{P-1}\, dl\, dm = H_\varpi(s,p)$$

$$\mathfrak{LM}\,[h(l,m)\, f(t-l,m)] = \int_0^\infty \int_0^\infty e^{(\pi p + \varpi^2 \log m - \varpi\ \log l) - sl}\ \ m^{P-1} h(l,m) f(t-l,m)\, dl\, dm$$

$$=\int_0^\infty\int_0^\infty f(t-l,m)\,e^{\alpha}\,m^{P-1}\left[\frac{1}{(2\pi i)^2}\int_{a-i\infty}^{a+i\infty}\int_{b-i\infty}^{b+i\infty}\frac{\pi^{\varpi}}{s^{\varpi}p}\;e^{sl+\pi p}\;m^{-p}\;\mathfrak{LM}\,[h(l,m)]\,dsdp\right]dldm$$

$$=\frac{1}{(2\pi i)^2}\int_{a-i\infty}^{a+i\infty}\int_{b-i\infty}^{b+i\infty}\frac{\pi^{\varpi}}{s^{\varpi}p}\;e^{sl+\pi p}\;m^{-p}\;\mathfrak{LM}\,[h(l,m)]\,dsdp\left[\int_0^\infty\int_0^\infty f(t-l,m)e^{\alpha}\,m^{P-1}\,dldm\right]$$

where , $(\pi p+\varpi^2\,log\,m-\varpi\;log\,l-sl)=\alpha$

$$=\frac{1}{(2\pi i)^2}\int_{a-i\infty}^{a+i\infty}\int_{b-i\infty}^{b+i\infty}\frac{\pi^{\varpi}}{s^{\varpi}p}\;e^{sl+\pi p}\,m^{-p}\;\mathfrak{LM}\,[h(l,m)]\;\mathfrak{LM}\,[f(t-l,m)]\;\;dsdp$$

so prove, when $\varpi=\frac{1}{2}$

$$\mathfrak{LM}\,[h(l,m)\,f(t-l,m)]=\frac{1}{(2\pi i)^2}\int_{a-i\infty}^{a+i\infty}\int_{b-i\infty}^{b+i\infty}\frac{\pi^{\varpi}}{s^{\varpi}p}\;e^{sl+\pi p}\,m^{-p}\;\mathfrak{LM}\,[h(l,m)]\;\mathfrak{LM}\,[f(t-l,m)]\,dsdp$$

when $\boldsymbol{\varpi}$=(0,-1,-2, … ,-n)

$$\mathfrak{LM}\,[h(l,m)\,f(t-l,m)]=\frac{1}{(2\pi i)^2}\int_{a-i\infty}^{a+i\infty}\int_{b-i\infty}^{b+i\infty}\frac{\Gamma(\varpi+1)}{s^{\varpi+1}p}\;e^{sl+\pi p}\,m^{-p}\;\mathfrak{LM}\,[h(l,m)]\;\mathfrak{LM}\,[f(t-l,m)]\,dsdp$$

4.3 ORTHOGONALITY THEOREM : When GLMIT is

$$\mathfrak{LM}\,[h(l,m)]=\int_0^\infty\int_0^\infty h(l,m)\;\;e^{(\pi p+\varpi^2\log m-\varpi\;\log l)-sl}\;\;m^{P-1}\,dl\,dm=H_{\varpi}(s,p)$$

then,

$$\mathfrak{LM}\,[h(l,m)\,f(t,m)]=\frac{1}{(2\pi i)^2}\int_{a-i\infty}^{a+i\infty}\int_{b-i\infty}^{b+i\infty}\frac{\pi^{\varpi}}{s^{\varpi}p}\;e^{sl+\pi p}\,m^{-p}\;\mathfrak{LM}\,[h(l,m)]\;\mathfrak{LM}\,[f(t,m)]\;\;dsdp$$

PROOF :

$$\mathfrak{LM}\,[h(l,m)]=\int_0^\infty\int_0^\infty h(l,m)\;\;e^{(\pi p+\varpi^2\log m-\varpi\;\log l)-sl}\;\;m^{P-1}\,dl\,dm=H_{\varpi}(s,p)$$

$$\mathfrak{LM}\,[h(l,m)\,f(l,m)]=\int_0^\infty\int_0^\infty e^{(\pi p+\varpi^2\log m-\varpi\;\log l)-sl}\;\;m^{P-1}h(l,m)f(l,m)\,dl\,dm$$

$$=\int_0^\infty\int_0^\infty f(l,m)\,e^{\alpha}\,m^{P-1}\left[\frac{1}{(2\pi i)^2}\int_{a-i\infty}^{a+i\infty}\int_{b-i\infty}^{b+i\infty}\frac{\pi^{\varpi}}{s^{\varpi}p}\;e^{sl+\pi p}\;m^{-p}\;\mathfrak{LM}\,[h(l,m)]\,dsdp\right]dldm$$

$$=\frac{1}{(2\pi i)^2}\int_{a-i\infty}^{a+i\infty}\int_{b-i\infty}^{b+i\infty}\frac{\pi^{\varpi}}{s^{\varpi}p}\;e^{sl+\pi p}\,m^{-p}\;\mathfrak{LM}\,[h(l,m)]\,dsdp\left[\int_0^\infty\int_0^\infty f(l,m)e^{\alpha}\,m^{P-1}\,dldm\right]$$

where , $(\pi p+\varpi^2\,log\,m-\varpi\;log\,l-sl)=\alpha$

$$=\frac{1}{(2\pi i)^2}\int_{a-i\infty}^{a+i\infty}\int_{b-i\infty}^{b+i\infty}\frac{\pi^{\varpi}}{s^{\varpi}p}\;e^{sl+\pi p}\,m^{-p}\;\mathfrak{LM}\,[h(l,m)]\;\mathfrak{LM}\,[f(l,m)]\;\;dsdp$$

so prove, *when* $\varpi=\frac{1}{2}$

$$\mathfrak{LM}\,[h(l,m)\,f(l,m)]=\frac{1}{(2\pi i)^2}\int_{a-i\infty}^{a+i\infty}\int_{b-i\infty}^{b+i\infty}\frac{\pi^{\varpi}}{s^{\varpi}p}\;e^{sl+\pi p}\,m^{-p}\;\mathfrak{LM}\,[h(l,m)]\;\mathfrak{LM}\,[f(l,m)]\;\;dsdp$$

when $\boldsymbol{\varpi}$=(0,-1,-2, … ,-n)

$$\mathfrak{LM}\,[h(l,m)\,f(l,m)]=\frac{1}{(2\pi i)^2}\int_{a-i\infty}^{a+i\infty}\int_{b-i\infty}^{b+i\infty}\frac{\Gamma(\varpi+1)}{s^{\varpi+1}p}\;e^{sl+\pi p}\,m^{-p}\;\mathfrak{LM}\,[h(l,m)]\;\mathfrak{LM}\,[f(l,m)]\;\;dsdp$$

4.4 FIRST SHIFTING THEOREM: When GLMIT is

$$\mathfrak{LM}\,[h(l,m)]=\int_0^\infty\int_0^\infty h(l,m)\;\;e^{(\pi p+\varpi^2\log m-\varpi\;\log l)-sl}\;\;m^{P-1}\,dl\,dm=H_{\varpi}(s,p)$$

then,

$$\mathfrak{LM}\,[e^{-al}m^{b}\;h(l,m)]=H_{\varpi}(s+a,p+b)$$
$$\mathfrak{LM}\,[e^{al}m^{-b}\;h(l,m)]=H_{\varpi}(s-a,p-b)$$

REFERENCES

[1] Zemanian A. H "Generalized Integral transformation", Inter science publisher, New York 1968.

[2] Zemanian A.H "The Distributional Laplace and Mellin transformations", J. SIAM , U.S.A, Vol. 14, Issue 1, 1908

[3] Ahmed I. Zayed "Handbook of Function and Generalized Function Transformations", CRC Press 1996.

[4] Sharma V.D, Thakare M.M "Generalized Laplace-Fractional Mellin Transform and operators", Int. J. Pure Appl. Sci. Technol.16(1), 2013 (pp.20-25).
[5] Sharma V.D. , Thakare M.M. "Generalized Laplace-Finite Mellin Transform", Int. Journal of Engineering Research and Applications, Vol.3,Issue 5 ,2013 (pp.1468-1470).
[6] Khairnar S.M. , Pise R.M. , Salunke J.N. "Bilateral Laplace-Mellin Integral Transform and its Applications" , Int. J. Pure Appl. Sci. Technol. , 1(2) 2010 (pp.114-126).
[7] Khairnar S.M. , Pise R.M. , Salunke J.N. "Relation of Finite Mellin Integral Transform with Laplace and Fourier Transforms" , Contemporary Engineering Sciences, Vol. 4, 2011 (pp.269-288).
[8] Khairnar S.M. , Pise R.M. , Salunke J.N. "Applications of the Laplace-Mellin Integral Transform to differential equations" , Int. J. of Scientific and Research Publication Vol. 2,2012.